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Static and Dynamic Analysis of Functionally Graded Materials

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Abstract— Functionally graded materials is an in-homogenous composite material characterized by a compositional gradient from one component to other which mainly constitutes chemical - alloy mixture of metals and ceramics. Within FGM the different microstructural phases have different functions, and overall FGM attain the multi-structural state with property gradient. By gradually varying volume fraction of constituent materials, the material properties exhibit a smooth and continuous change across the thickness. We present several numerical simulations, in order to assess the behavior of functionally graded plates subjected to mechanical loads and natural frequency. A simple supported plate and fixed-fixed plate is considered for the investigation. The plate is made up of a ceramic material at the top, a metallic at the bottom. The simple power law with different values of n (volume coefficient) is used through the thickness variation and all results are tabulated and necessary graphs are drawn.

1 INTRODUCTION

Functionally significant properties like mechanically brittle, good temperature behaviour, high corrosion resistance required for engineering materials. In FGM, the toughness of a metal can be mated with the refractoriness of a ceramic, without any compromise in the toughness of the metal side or the refractoriness of the ceramic side. For example, in Rocket heat shields, fusion reactors the material is coated with ceramic on one side which is exposed to high temperature, whereas on other side it is coated with metal which is exposed to cold region. Living tissues like bones and teeth are characterized as natural form of FGM. Even our skin is also graded to provide certain toughness, tactile and elastic qualities as a function of skin depth and location on the body. On damage of tissues like bones and teeth, it is replaced with bio-compatible material which serves the purpose of original bio-tissue. FGM serves best in this aspect. Hence FGM exhibits wide range of applications of dental and orthopaedic applications for teeth and bone replacement.

Applications of FGM are

1)FGM coating and interface can be used to reduce the residual stress and thermal stress

2)FGM as an interface layer to connect two incompatible materials can greatly enhance the bond strength

3)FGM coating reduce the crack driving force as perfect interlocking of layers takes place.

1.2 Laws of FGM

The optimum material gradient of a rectangular plate made of functionally graded materials (FGM) is determined in this study. Elastic modulus of functionally grade rectangular plate is assumed to vary continuously throughout the thickness of the plate, according to the volume fraction of the constituent materials based on the power law, exponential law and sigmoid law. For the change in material properties of the FGM the numerical method is used to break the entire FGM into various layers; in order to capture the change in properties. These layers capture a finite portion of the thickness and are treated like isotropic materials. Materials properties are calculated from the bottom surface using the various volume fraction distribution. The layers and their associated properties are then layered together to establish through the thickness variation of material properties.

2 LITERATURE SURVEY

[1] Manish Bhandari and Dr Kamlesh Purohit

They have studied that by varying volume fraction distributions and boundary conditions. Also static analysis of functionally gradient material plate is carried out by sigmoid law and verified with the published results. The convergence study of the results is optimized by changing the mesh size and layer size. Power law and exponential law are applied for the same material and set of conditions.

[2] Vyacheslav N. Burlayenko Tomasz Sadowski

This paper is based on model and static analyses. Demonstrated for square metal-ceramic FG simply supported plates through-the-thickness variation of the volume fraction of the ceramic constituent.

[3] MaarjusKirsa, KristoKarjusta, Imran Azizb, ErkoOunapuua, and Ernst Tungela

This paper is based on the evaluation of formulation-based methods (FDM and DQM). A solid element 3D finite element model is developed and the numerical results obtained by using simplified approaches are confirmed.

[4] Paul , D Das

They investigated that the dynamic problem for the pre-stressed beam which has clamped–clamped, simply supported–simply supported and clamped–simply supported boundary conditions.

Four different FGM beams, namely Stainless Steel– Silicon Nitride, Stainless Steel–Zirconia, Stainless Steel–Alumina and Titanium alloy–Zirconia, are considered for generation of results.

2.1 OBJECTIVES

- Static analysis of FG beams and plates using ANSYS
- Free vibration analysis of FG beams and plates using ANSYS
- Comparison of FE results with results obtained from literature
- Analyzing material behavior of Y-TZP Ceramics, used in clinical applications and its effectiveness for case study

3 METHODOLOGY

The flowchart shows the methodology for the simulation of functionally graded plates.

Static Analysis of beams and plates

↓

Free vibrational analysis in beams

↓

Matlab analysis to find material properties of FGM layer

↓

Comparison of FGM layer plate with single layer plate

4 ANALYSIS AND CALCULATIONS

4.1 Analysis of Isotropic beams

FGM properties are mainly viewed in terms of aspect ratio (ratio of width to height).Hence, we took height(b) equal to 50mm for all cases and changed width(a) values to get different aspect ratios like 0.5,1,2,4,6,8,10 and length of beam is taken as 1000mm, beam material taken as Structural Steel for standard purpose.

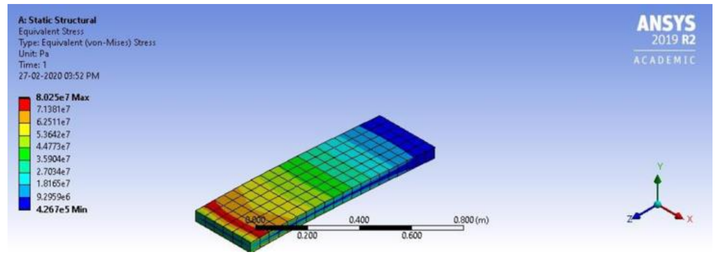

Fig 4.1: Deflection of cantilever beam (a=50mm, b=50mm) of aspect ratio equal to 1.

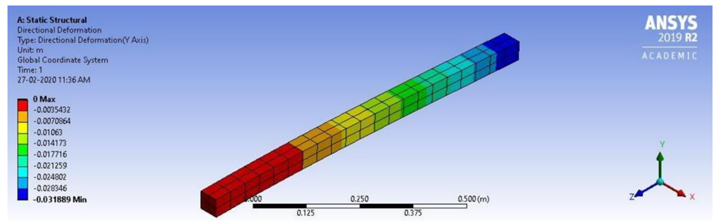

Fig 4.2: Stress of cantilever beam (a=300mm, b=50mm) of aspect ratio equal to 6.

Aspect ratio	Deflection (m)	Stress(N/m^2) × 10^8
0.5	0.0025183	1.1992
1	0.0012596	0.59969
2	0.00063002	0.3004
4	0.0003165	0.15043
6	0.00021237	0.10066
8	0.00016031	0.0075701
10	0.0001293	0.0061088

Table 4.1: Deflection and Stress of Cantilever beam with Point load at free end.

Aspect ratio	Deflection (m)	Stress(N/m^2)× 10^8
0.5	0.063872	9.5794
1	0.031889	4.8815
2	0.015829	2.4857
4	0.0078451	1.2174
6	0.005185	0.08025
8	0.0038583	0.060743
10	0.0030667	0.048615

Table 4.2: Deflection and Stress of Simply supported beam with uniformly distributed load.

Aspect ratio	Deflection (m)	Stress(N/m^2) ×10^8
0.5	0.063872	9.5794
1	0.031889	4.8815
2	0.015829	2.4857
4	0.0078451	1.2174
6	0.005185	0.08025
8	0.0038583	0.060743
10	0.0030667	0.048615

Table 4.3: Deflection and Stress of Fixed-Fixed beam with uniformly distributed load.

From the above tables, it is observed that, as the aspect ratio increases both the stress and deflection decreases.

4.2 Modal analysis of isotropic beams

Structures can be designed using static analysis but they will have to be overly conservative to be safe. Hence dynamic analysis is carried. First mode of natural frequency and second mode of natural frequency are found to cantilever beam, simply supported beam, fixed-fixed beam. Width(a) is taken 50mm for all cases and changed height(b) values to get different aspect ratios like 0.5,1,2,4,6,8,10 and length of beam is taken as 1000mm, beam material taken as Structural Steel for standard purpose.

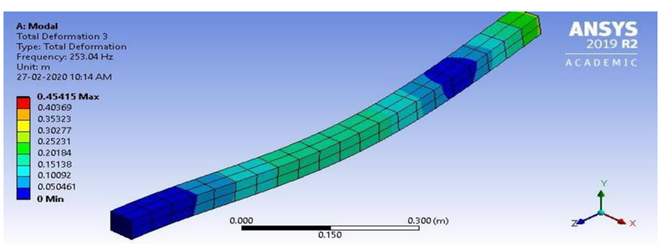

Figure 4.3: Cantilever beam first mode of natural frequency for aspect ratio equal to 1

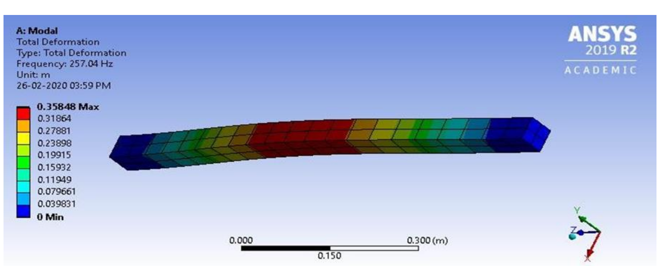

Figure 4.4: Fixed- Fixed beam with first mode of natural frequency for aspect ratio equal to 1

Aspect ratio	First mode of natural frequency (Hz)	Second mode of natural frequency (Hz)	Third mode of natural frequency (Hz)
0.5	20.4	128	358
1	40.8	256	715
2	81.5	511	1430
4	163	1020	2860
10	408	2560	7150

Table 4.4: Results of modal analysis for fixed-fixed beam for different aspect ratios.

Aspect ratio	First mode of natural frequency (Hz)	Second mode of natural frequency (Hz)	Third mode of natural frequency (Hz)
0.5	57.2	229	515
1	114	458	1030
2	229	916	2060
4	458	1830	4120
8	916	3660	8240
10	1140	4580	10300

Table 4.5: Results of modal analysis for cantilever beam for different aspect ratios.

Aspect ratio	First mode of natural frequency (Hz)	Second mode of natural frequency (Hz)	Third mode of natural frequency (Hz)
0.5	130	290	604
1	259	580	1210
2	519	1160	2420
4	1040	2320	4830
8	2080	4640	9670
10	2590	5800	12100

Table 4.6: Results of modal analysis for simply supported beam for different aspect ratios.

From the above tables, it is observed that all the modes of natural frequencies increase with increase in the aspect ratios.

4.3 Static Analysis of isotropic plates

The FGM plate made of combination of metal and ceramic is taken for static and dynamic analysis. The metal and ceramic chosen are aluminium and alumina respectively. Young's modulus for aluminium is 70 GPa and that for alumina is 380 GPa. Poisson's ratio for both of the materials was chosen to be 0.3 for simplicity. The FGM plate has taken parameters like thickness for the plate(h) as 0.01m and one of the side length(a) is taken as 0.1m(a=10h), changing the value of b (other side length), which results in change of aspect ratio. The mechanical analysis was performed by applying uniformly distributed load (UDL) with varying aspect ratio(a/b). The value of uniformly distributed load is taken as 0.01 MPa.

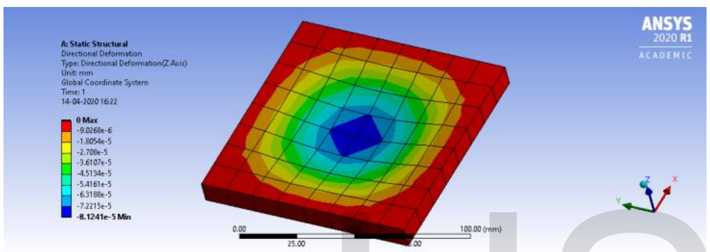

Figure 4.5: Directional Deformation (Z axis).

4.4 **Static Analysis of FGM plates**

The FGM plate made of combination of metal and ceramic is taken for static and dynamic analysis. The metal and ceramic chosen are aluminium and alumina respectively. Young's modulus for aluminium is 70 GPa and that for alumina is 380 GPa. Poisson's ratio for both of the materials was chosen to be 0.3 for simplicity. The FGM plate has taken parameters like thickness for the plate(h) as 0.01m and one of the side length(a) is taken as 0.1m(a=10h), changing the value of b (other side length), which results in change of aspect ratio. The mechanical analysis was performed by applying uniformly distributed load (UDL) with varying aspect ratio(a/b). The value of uniformly distributed load is taken as 0.01 MPa.

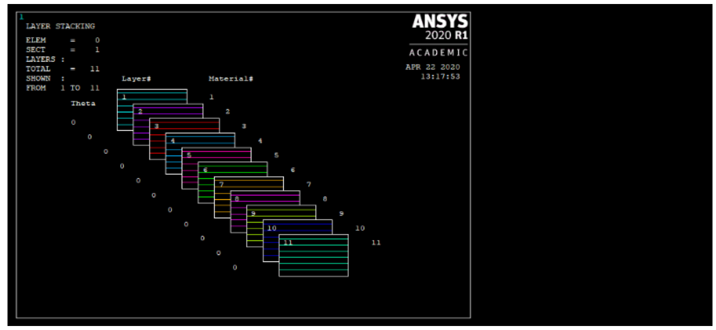

Figure 4.6: FGM plate OF 11 Layers

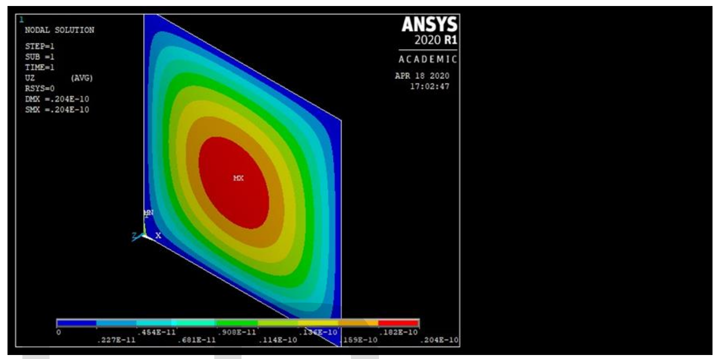

Figure 4.7: Directional Deformation (Z axis) of FGM plate aspect ratio equal to 1.

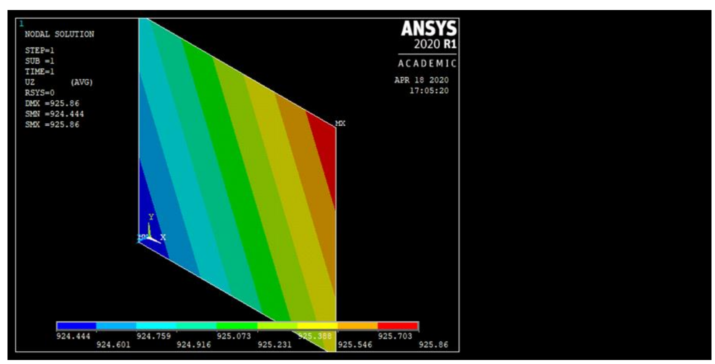

Figure 4.8: Directional Deformation (Z axis) of FGM plate aspect ratio equal to 1

Aspect ratio (a/b)	Maximum normal stress of FGM plate	Maximum deflection of FGM plate (Z axis)
0.5	454.85×10^{3}	1.59×10^{-7}
1	240.94×10^{3}	2.32×10^{-8}
2	11.478×10^{3}	1.94×10^{-8}
5	23.292×10^{3}	1.35×10^{-11}
8	0.92	1.35×10^{-12}
10	0.58	3.47×10^{-13}

Table 4.7: Maximum normal stress and deflection of Fixed-Fixed FGM plate for different aspect ratios

Aspect ratio (a/b)	Maximum normal stress	Maximum deflection of aluminum (Z axis)	Maximum deflection of alumina (Z axis)
0.5	495.17×10^{3}	3.92×10^{-6}	7.23×10^{-8}
1	308.07×10^{3}	1.97×10^{-7}	3.63×10^{-8}
2	12.23×10^{3}	1.18×10^{-8}	2.15×10^{-9}
5	57.35	1.23×10^{-10}	2.26×10^{-11}
8	3.06	1.17×10^{-11}	2.16×10^{-12}
10	0.8	3.84×10^{-12}	7.08×10^{-13}

Table 4.8: Maximum normal stress and deflection of Fixed-Fixed plate for different aspect ratios

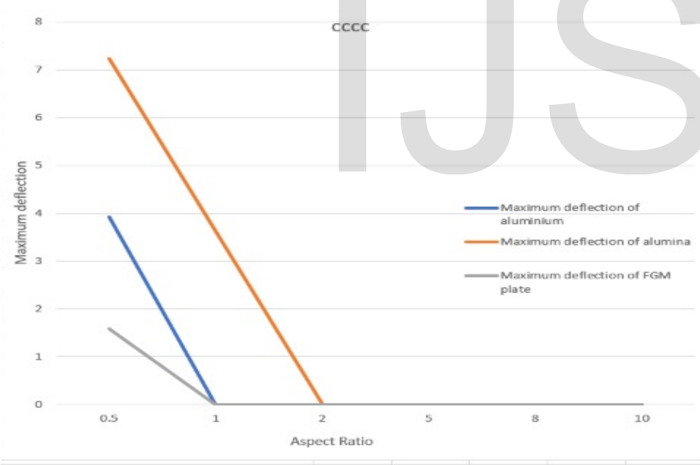

Figure 4.9: Maximum deflection v/s Aspect ratio of fixed all sides plate

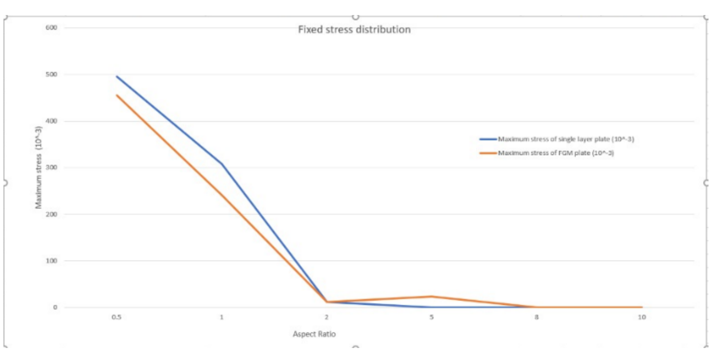

Figure 4.10: Maximum stress v/s Aspect ratio of fixed all sides plate.

From the above graph it can be noted that deflection of aluminium(red) which is very high. The deflection of alumina(blue) is less compared to the aluminium. The deflection of the FGM is the minimum.

4.5 Analysis of isotropic plates

Matlab analysis is done to find the variation of Youngs modulus along thickness using P-Law of FGM for various values of n.

- z_mat=linspace(-0.005,0.005,17);
- p_mat=zeros(1,length(z_mat));
- n_mat=[0,0.1,0.2,0.5,1,2,5,10,100];
- h=0.01;
- n=0.5;
- pt=380e9;
- pb=70e9;
- for i=1:length(z_mat)
- z=z_mat(i);
- p=((pt-pb)*(((z/h)+0.5)^n))+(pb);
- p_mat(i)=p;
- end
- plot(z_mat,p_mat);
- title("P-FGM");
- xlabel("Thickness(z)");
- ylabel("Young's modulus");

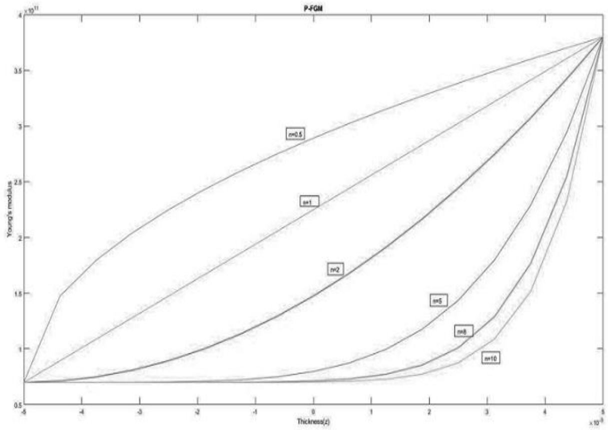

Figure 4.11: Youngs modulus v/s thickness using P-Law of FGM for various values of n

4.6 Modal Analysis of FGM plates

The FGM plate has taken parameters like thickness for the plate(h) as 0.01m and side lengths are taken as 0.1m.

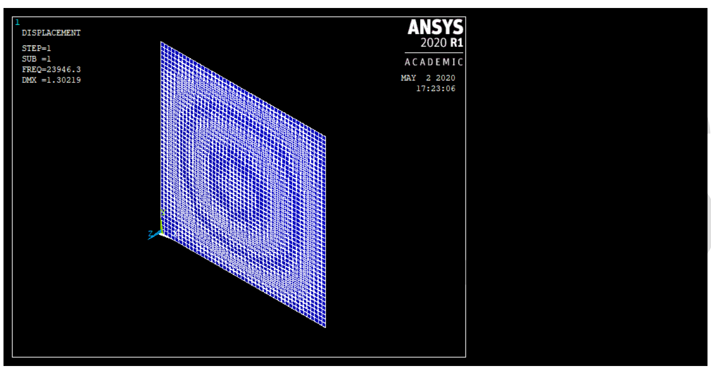

Figure 4.12: Fixed on all sides of plate, First mode of natural frequency of FGM plate aspect ratio equal to 1

Mode of Natural frequency	frequency (Hz)
1	23946
2	35809
3	35809
4	39152

Table 4.9: FGM plate natural frequency values with aspect ratio equal to 1 for fixed on all sides plate

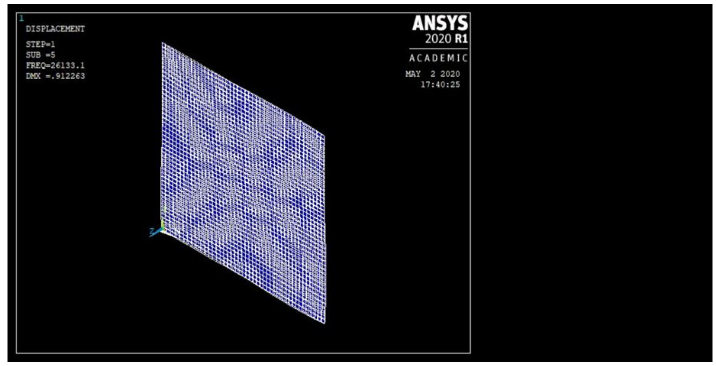

Figure 4.13: Simply supported on all sides of plate, Second mode of natural frequency of FGM plate aspect ratio equal to 1

Mode of Natural frequency	frequency (Hz)
1	0
2	0
3	0
4	20096

Table 4.10: FGM plate natural frequency with aspect ratio equal to 1 for simply supported on all sides plate.

5 Result

In this section we present several numerical simulations, in order to assess the behavior of functionally graded plates subjected to mechanical loads and natural frequency. A simple supported plate and fixed-fixed plate is considered for the investigation. The plate is made up of a ceramic material at the top, a metallic at the bottom. The simple power law with different values of n (volume coefficient) is used for the through the thickness variation and all results are tabulated and necessary graphs are drawn.

6 Conclusion

In the present work, finite element analysis is carried out on a functionally gradient material plate made of Aluminum/Alumina. The plate considered is thick plate with a/h=10 and a/b=0.5,1,2,5,8,10. The structural response of this plate is studied with respect to mechanical loads and compared with pure metal and pure ceramic plate under mechanical loading. The properties of functionally gradient material are calculated for each layer according to power law. The number of layers is kept constant. The following points are summarized:

a) Mechanical deformation of functionally graded ceramic-metal plates with varying aspect ratio is analyzed. It is observed that the bending response of the functionally graded plate is intermediate to those of the metal and the ceramic plates.

b) ANSYS gives faster results and degree of accuracy depends on the mesh size, layers and solver.

c) The numeric and analytic values of mechanical properties of the structures are tabulated, which are mimic the behavior of dental implants.

7 FUTURE SCOPE

We recommend further investigation of functionally graded plate structures with material properties varying in direction other than through the thickness. A further investigation regarding the techniques for estimating effective material properties of functionally graded materials is desirable. In the graded layer of real FGMs, ceramic and metal particles of arbitrary shapes are mixed up in arbitrary dispersion structures. Hence, the prediction of thermo-elastic properties is not a simple problem, but complicated due to the shape and orientation of particles dispersion of structure and volume fraction. The thickness of the middle FGM can be optimized.

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